Giovanni Anastasio

tq- System Problem Set

1. Write down the axiom schema and the three shortest axioms in the tq- system.
    a. Axiom schema- xt-qx is an axiom, whenever x is a hyphen-string.
    b. 3 shortest axioms-
        i. -t-q-
        ii. -t--q--
        iii. -t---q---
2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string: -----t-----q-----.
    a. The sole rule of inference: suppose that x, y, and z are all hyphen strings. And suppose that xtyqz is an old theorem. Then, xty-qzx is a new theorem.
        i. -----t-----q----- → -----t------q------
3. Reasoning in I-mode argue that the string you produced in the previous item is not a theorem in the tq- System.
    a. The previous item is not a theorem because the first two hyphen groups do not multiply to equal the third hyphen group.
4. Working in M-mode show that -----t---q--------------- is a theorem in the tq- System.
    a. -----t-q-----   Axiom
    b. -----t--q----------   From a
    c. -----t--q---------------   From b
5. What are the two rules of the C- System?
    a. The two rules of the C- system are:
        i. Suppose x, y, and z are hyphen strings. If x-ty-qz is a theorem then Cz is a theorem.
        ii. Suppose that x is a hyphen string. If Cx is not a theorem, then px is a theorem.
6. Working within the C-system, argue that c-------- is a theorem of the system.
    a. ----t-q----   axiom
    b. ----t--q--------   From a by rule of inference
    c. C--------   From b by rule of C systems
7. Does adding the following rule to the C- system constitute a Post production system for determining primes? Suppose x is a hyphen-string. If Cx is not a theorem, then px is a theorem. Explain.
    a. Yes because it will indicate which numbers are prime and if they are not indicated as prime they are composite and vice verse.
8. Hofstadter writes: When a figure or “positive space” (eg., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape - also called the “ground”, or “background”, or “negative space” - has also been drawn. Explain how the picture is considered negative space.
    a. The picture of the quiche pan shown has negative space all around the border of the figure, showing the figures true shape

9. Answer the following questions with respect to the A-System:
    a. Show that A-------- is a theorem of the A- System by working within the system.
        i. A-- Axiom
        ii. A---- From (1)
        iii. A------ From (2)
        iv. A-------- From (3)
    b. Specify a decision procedure for determining theorem hood in the A- System.
        i. There needs to be an even number of hyphens after the A.
    c. Provide an I-mode argument that the string A----------- is not a theorem of the A-System.
        i. The string A----------- is not a theorem of the A- System because there is not an even number of hyphens after the A.
    d. What subset of the natural numbers do you think it was my intent to capture with the A- System?
        i. Even numbers
10. Consider the as yet to be formally defined B-System which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture.
    a. Propose, by analog with the rule on page 66 of GEB, an invalid rule for producing theorems in the B- System.
        i. An invalid rule for producing theorems in the B- system is to require an even number of hyphens follow B.
    b. Define a (valid) Post production system for the B- System in terms of one axiom and one rule.
        i. B-
        ii. There needs to be an odd number of hyphens after the B.
    c. Derive B----------- within the B- System.
        i. B- Axiom
        ii. B--- From (1)
        iii. B----- From (2)
        iv. B------- From (3)
        v. B--------- From (4)
        vi. B----------- From (5)
    d. What subset of the natural numbers does the B- System capture?
        i. Odd numbers
11. Under interpretation, what does the A- System A-------- say? Under interpretation, what does the B- System B-------- say?
    a. Under interpretation, the A-System says that to be a theorem you must have an even number of hyphens within the theorem. Under interpretation, the B-System says that to be a theorem you must have an odd number of hyphens within the theorem
12. What does it mean for a set to be “recursively enumerable”? What does it mean for a set to be “recursive”?

    a. For a set to be recursively enumerable means it can be generated according to typographical rules. For example: the set of C type theorem, the set of theorems of the MIU System, the set of theorems of any formal system. For a set to be recursive is like a figure whose ground is also a figure. The set is recursively enumerable, and its compliment is also recursively enumerable.

13. Argue that the set of even numbers is **recursively enumerable**.
    a. The set of even numbers is recursively enumerable because the set can be generated according to typographical rules like in the A- System.
14. Argue that the set of even numbers is **recursive**.
    a. The set of even numbers is recursive because the set of even numbers itself is recursively enumerable and its compliment the set of odd numbers is also recursively enumerable.
15. Argue that the set of prime numbers is **recursively enumerable**.
    a. The set of even numbers is recursively enumerable because the set can be generated according to typographical rules like in the C- System.
16. Argue that the set of prime numbers is **recursive**.
    a. The set of even numbers is recursive because the set of prime numbers itself is recursively enumerable and its compliment the set of composite numbers is also recursively enumerable.
17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P----- within the system
18. Answer the following questions about the @|- system.
    a. How many axioms in the system? How many rules in the System? How many Theorems in the system?
        i. There is one axiom, 34 rules, and infinite number of theorems in the system.
    b. Show that @| | - | | | - | | - | | | - is a theorem of the @|- system by performing a derivation within the System.
        i. @| | - | | | - (Axiom)
        ii. @| | - | | | - | (Rule 6)
        iii. @| | - | | | - | | (Rule 4)
        iv. @| | - | | | - | |- (Rule 2)
        v. @| | - | | | - | | - | (Rule 7)
        vi. @| | - | | | - | | - | | (Rule 5)
        vii. @| | - | | | - | | - | | | (Rule 6)
        viii. @| | - | | | - | | - | | | - (Rule 1)
    c. Show that CDEFGABC is a theorem of the @|- System by performing a derivation within the system.
        i. @| | - | | | - (Axiom)
        ii. C| | - | | | - (Rule 8)
        iii. CD | - | | | - (Rule 11)
        iv. CDE - | | | - (Rule 15)
        v. CDEF | | | - (Rule 20)

vi. CDEFG | | - (Rule 21)
vii. CDEFGA | - (Rule 25)
viii. CDEFGAB - (Rule 29)
ix. CDEFGABC (Rule 34)

d. Show that GABCDEXG is a theorem of the @|- System by performing a derivation within the system.
   i. @| | - | | | - (Axiom)
   ii. G| | - | | | - (Rule 9)
   iii. GA | - | | | - (Rule 25)
   iv. GAB - | | | - (Rule 29)
   v. GABC | | | - (Rule 34)
   vi. GABCD | | - (Rule 11)
   vii. GABCDE | - (Rule 15)
   viii. GABCDEX - (Rule 19)
   ix. GABCDEXG (Rule 24)
e. Show that DEXGABVD is a theorem of the @|- System by performing a derivation within the system.
   i. @| | - | | | - (Axiom)
   ii. D| | - | | | - (Rule 10)
   iii. DE | - | | | - (Rule 15)
   iv. DEX - | | | - (Rule 19)
   v. DEXG | | | - (Rule 24)
   vi. DEXGA | | - (Rule 9)
   vii. DEXGAB | - (Rule 29)
   viii. DEXGABV - (Rule 33)
   ix. DEXGABVD (Rule 14)
f. What do you think I had in mind when I invented this system?
   i. What I believe you had in mind when you invented this system is to show us an example of a system that did not just consist of a letter or two with a small number of rules. You wanted to show that a system could be composed of multiple different character strings with various rules to follow.